Exercices

1. The group G is co-Hopfian if any injective homomorphism $G \to G$ has to be an isomorphism.
   (1) The groups $\mathbf{Q}$, $\mathbf{Q}/\mathbf{Z}$ are co-Hopfian.
   (2) Prove that the fundamental group of a surface of genus g is co-Hopfian.
   (3) Prove that $\mathbf{Z}^{\mathbf{k}}$, free non-abelian groups, $\mathbf{Q}^*$, $\mathbf{R}$ are not co-Hopfian.
   (4) Prove that $PSL(2, \mathbf{Z})$ is not co-Hopfian.

2. A group acting faithfully on a rooted tree is residually finite.
   (1) Conversely, any finitely generated residually finite group acts faithfully on a locally finite rooted tree.
   (2) A group is called residually p-finite if any nontrivial element of it can be detected by representing the group into a finite p-group. Show that a finitely generated residually p-finite group acts faithfully on a rooted homogeneous tree of degree p.

3. A finitely generated torsion free metabelian group is residually finite.