Interrogation de calcul 12

Question 1. Calculer le $DL_5(0)$ de $x \mapsto \tan(x)\sin(x)$.

$$\tan(x)\sin(x) = \left(x + \frac{x^3}{3} + o(x^4)\right)\left(x - \frac{x^3}{6} + o(x^4)\right)$$

$$= x^2 - \frac{x^4}{6} + o(x^5)$$

$$+ \frac{x^4}{3}$$

$$= x^2 + \frac{x^4}{6} + o(x^5),$$

Question 2. Calculer le $DL_8(0)$ de $x \mapsto \tan(x)\cos(x) = \sin(x)$!!

$$\tan(x)\cos(x) = x - \frac{x^3}{6} + \frac{x^5}{120} - \frac{x^7}{5040} + o(x^8).$$

Question 3. Calculer le $DL_4(0)$ de $x \mapsto \ln(\cos(2x))$.

$$\cos(2x) = 1 - \frac{(2x)^2}{2} + \frac{(2x)^4}{24} + o(x^4)$$

$$= 1 - 2x^2 + \frac{2}{3}x^4 + o(x^4)$$

donc $\ln(\cos(2x)) = \ln\left(1 - 2x^2 + \frac{2}{3}x^4 + o(x^4)\right)$

$$= \left(-2x^2 + \frac{2}{3}x^4 + o(x^4)\right) - \frac{1}{2}\left(-2x^2 + o(x^2)\right)^2 + o(x^4)$$

$$\text{car} \begin{cases} \ln(1+u) = u - \frac{u^2}{2} + o(u^2) \\ -2x^2 + \frac{2}{3}x^4 + o(x^4) \sim -2x^2 \end{cases}$$

$$= -2x^2 + \frac{2}{3}x^4 - 2x^4 + o(x^4)$$

$$= -2x^2 - \frac{4}{3}x^4 + o(x^4).$$

Question 4. Calculer le $DL_2(0)$ de $x \mapsto \dfrac{1}{e^x + \cos(x)}$.

$$\frac{1}{e^x + \cos x} = \left(\left(1 + x + \frac{x^2}{2} + o(x^2)\right) + \left(1 - \frac{x^2}{2} + o(x^2)\right)\right)^{-1}$$

$$= \left(2 + x + o(x^2)\right)^{-1}$$

$$= \frac{1}{2}\left(1 + \frac{x}{2} + o(x^2)\right)^{-1}$$

$$= \frac{1}{2}\left[1 - \left(\frac{x}{2} + o(x^2)\right) + \left(\frac{x}{2} + o(x)\right)^2 + o(x^2)\right]$$

$$\text{on} \begin{cases} (1+u)^{-1} = 1 - u + u^2 + o(u^2) \\ \frac{x}{2} + o(x^2) \sim \frac{x}{2} \end{cases}$$

$$= \frac{1}{2}\left[1 - \frac{x}{2} + \frac{x^2}{4} + o(x^2)\right]$$

$$= \frac{1}{2} - \frac{x}{4} + \frac{x^2}{8} + o(x^2).$$